Numerical optimization of tokamak and stellarator equilibrium

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Theory Department Research and Review Seminar

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PPPL U.S. DEPARTMENT OF ENERGY

Before we begin some background

- Funding levels (2015)
    - 70% W7-X collaboration
    - 20% NSTX-U/Theory partnership
    - 10% LDRD
- Leaving for W7-X for 9? months in March
- Things I'm not talking about today
    - VMEC Benchmarking
    - DIAGNO Benchmarking
    - SPEC ITER/DIII-D Calculations
    - W7-X Field line mapping experiments

Overview of my optimization work

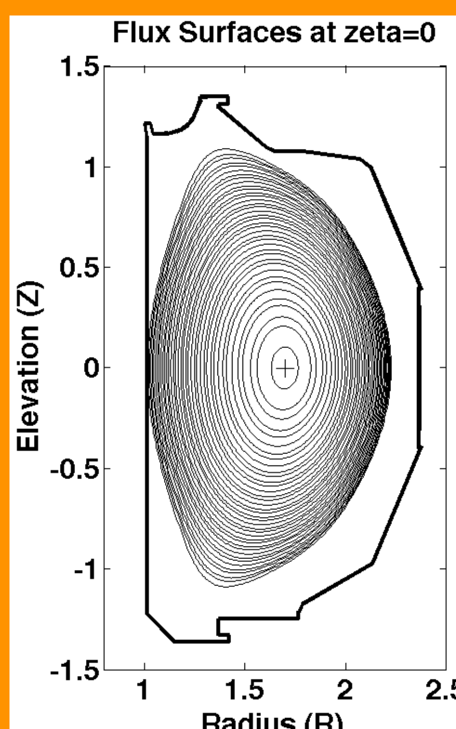
- LHD Equilibrium Reconstruction
    - Initial work with STELLOPT and VMEC
    - Rewrite and validation of magnetic diagnostics code (DIAGNO)
    - MSE synthetic diagnostics into STELLOPT
- Tokamak 3D Equilibrium Reconstruction
    - Rewrite of STELLOPT (STELLOPTV2)
    - DIII-D Reconstruction (NE,TE,TI profile diagnostics)
    - ITER forward modeling of applied RMP's
- Turbulent Transport Optimization
    - Implementation of parallel GENE in STELLOPTV2
- Development of IPECOPT for NTV optimization on NSTX-U

Optimization of equilibrium in a nutshell.

Input Parameters
- Currents
- Vacuum Fields
- Pressure, etc.

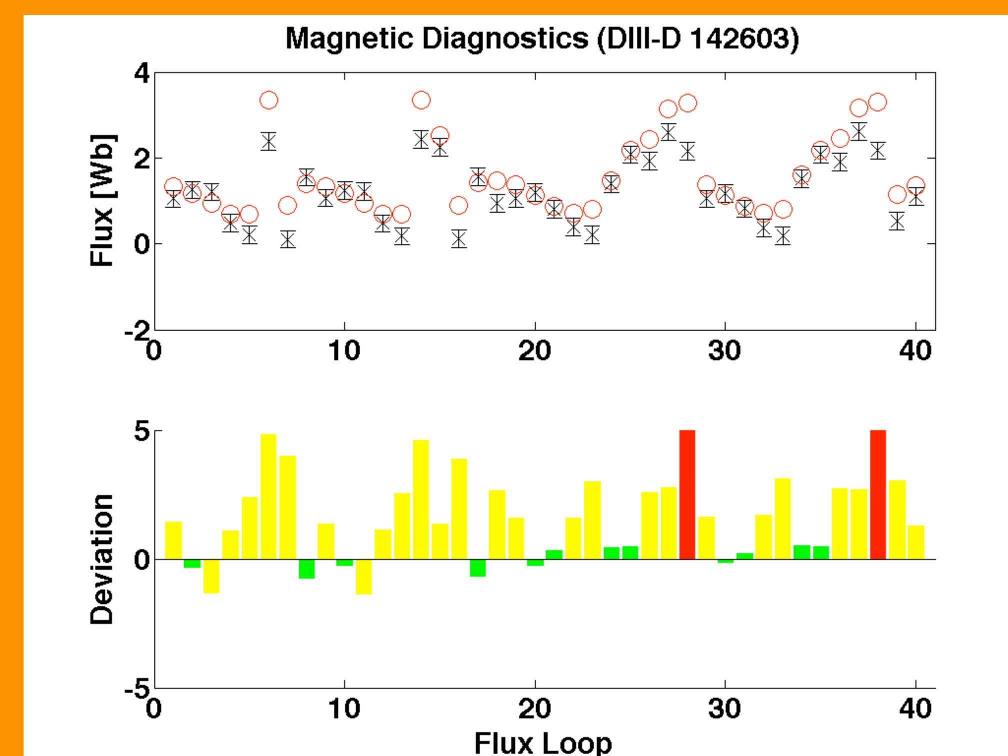
These need to result in a good match to these.

Equilibrium Model



Synthetic Signals

Measurements



Mathematically how can this be achieved?

- Begin with some function we wish to minimize (optimize)

$$y_j(p_i): \text{Function}$$

$$p_i: \text{Parameters}$$

- Given a set of parameters how do we choose better ones?

$$\tilde{p}_i = p_i + h_i$$

Mathematically how can this be achieved?

- Begin with some function we wish to minimize (optimize)

$$y_j(p_i) : \text{Function}$$

$$p_i : \text{Parameters}$$

In practice we minimize

$$\chi^2(p_i) = \sum_j \left| \frac{Y_j - y_j(p_i)}{\sigma_j} \right|^2$$

- Given a set of parameters how do we choose better ones?

$$\tilde{p}_i = p_i + h_i$$

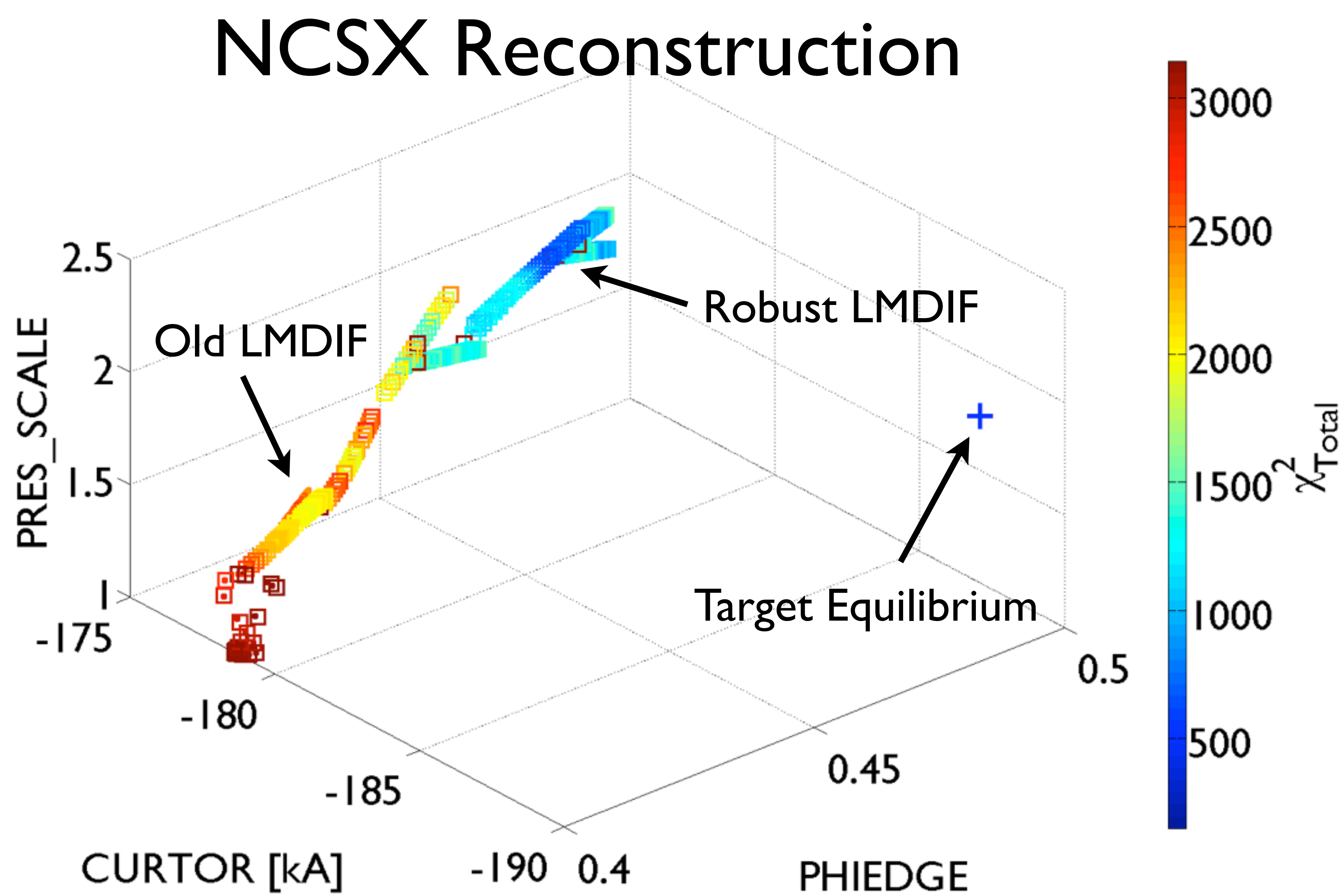
Numerical optimization methods are numerous

- Derivative Methods $\vec{p}_f = \vec{p}_0 + \vec{h}$
  - Gradient Descent $\vec{h}_{GD} = \alpha \tilde{J}^{\dagger} \tilde{W} (\vec{y}_{target} - \vec{y}(\vec{p}))$
  - Gauss Newton $\vec{h}_{GN} = [\tilde{J}^{\dagger} \tilde{W} \tilde{J}]^{-1} \tilde{J}^{\dagger} \tilde{W} (\vec{y}_{target} - \vec{y}(\vec{p}))$
  - Levenberg $\vec{h}_{LEV} = [\tilde{J}^{\dagger} \tilde{W} \tilde{J} + \lambda \tilde{I}]^{-1} \tilde{J}^{\dagger} \tilde{W} (\vec{y}_{target} - \vec{y}(\vec{p}))$
  - <u>Levenberg-Marquardt</u> $\vec{h}_{LEV-MAR} = [\tilde{J}^{\dagger} \tilde{W} \tilde{J} + \lambda diag(\tilde{J}^{\dagger} \tilde{W} \tilde{J})]^{-1} \tilde{J}^{\dagger} \tilde{W} (\vec{y}_{target} - \vec{y}(\vec{p}))$
- <u>Genetic Algorithms</u>
- <u>Particle Swarm</u>
- Simulated Annealing

The modified Levenberg-Marquardt

- Parallelized over Jacobian evaluation
- Scan over the Levenberg parameter
- STEP_OPT procedure if Jacobian evaluation yields better results than Levenberg step (M. Zarnstorff)
- Failed Jacobian directions removed from the analysis (Robust Jacobian)

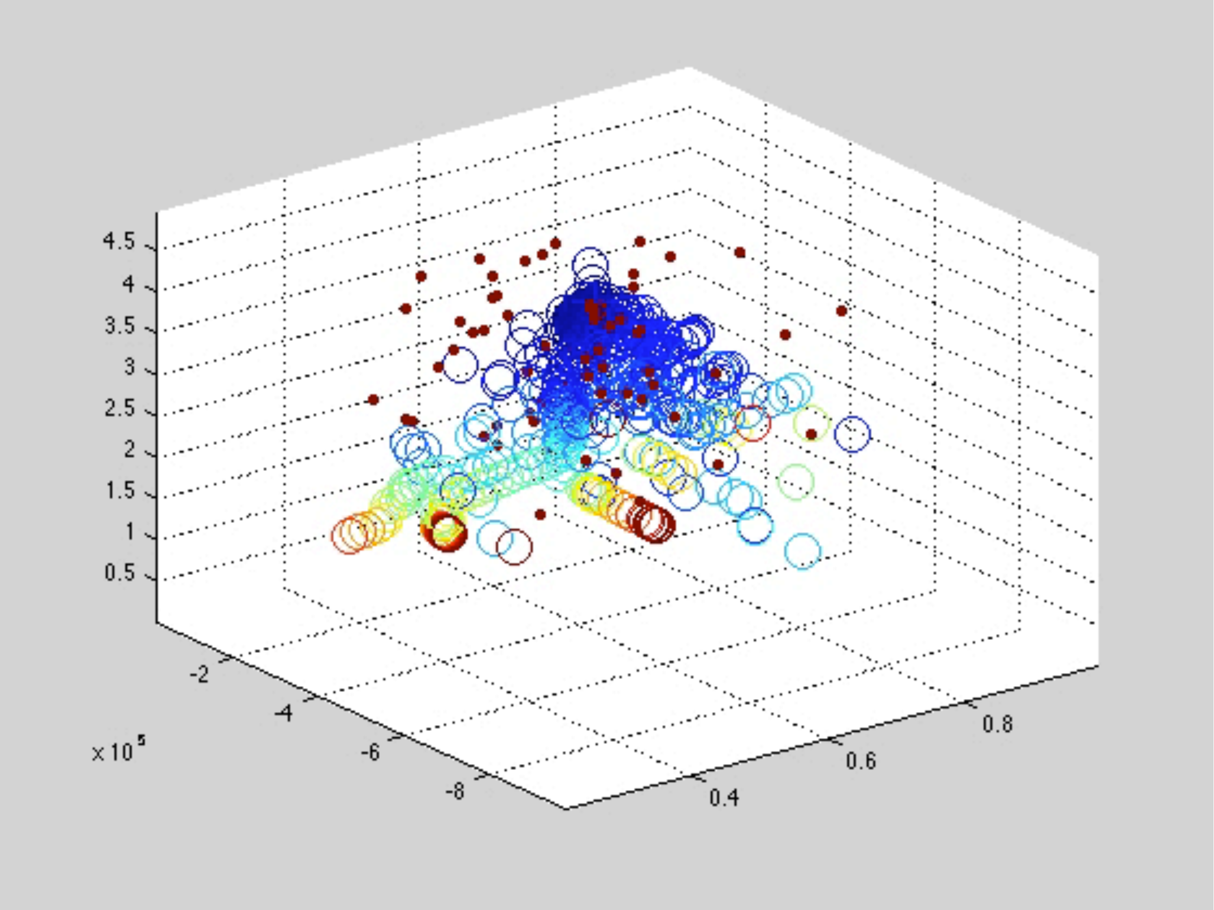
Robust Jacobian improves fit



Approximately 1/2 of function evaluations forced to fail.

Particle Swarm Optimization (PSO)

- Initial random distribution of points in the parameter hyperspace.
- Each particle has a unique velocity through hyperspace.
- Each particle has a record of its best solution
- All particles keep track of the overall best solution found



Velocity Equation

$$\vec{v}_i^{\,new} = \vec{v}_i + C_1\left(\vec{x}_i^{\,best} - \vec{x}_i\right) + C_2\left(\vec{x}_{global}^{\,best} - \vec{x}_i\right)$$

Position Update

$$\vec{x}_i^{\,new} = \vec{x}_i + \vec{v}_i$$

Eberhart, R. & Kennedy, J., "A new optimizer using particle swarm theory," *Proc. of the Sixth Intl. Sym. on Micro Machine and Human Science*. Oct 1995

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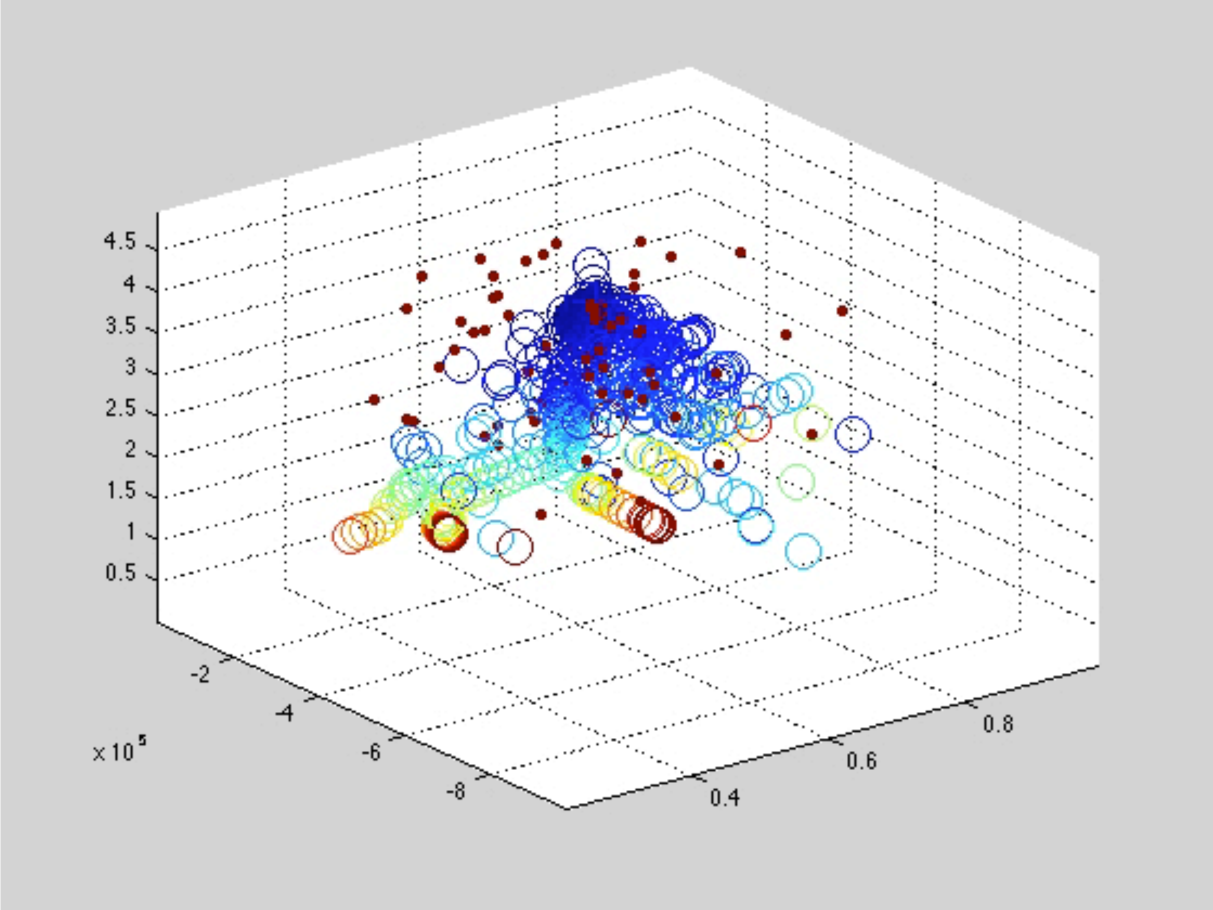

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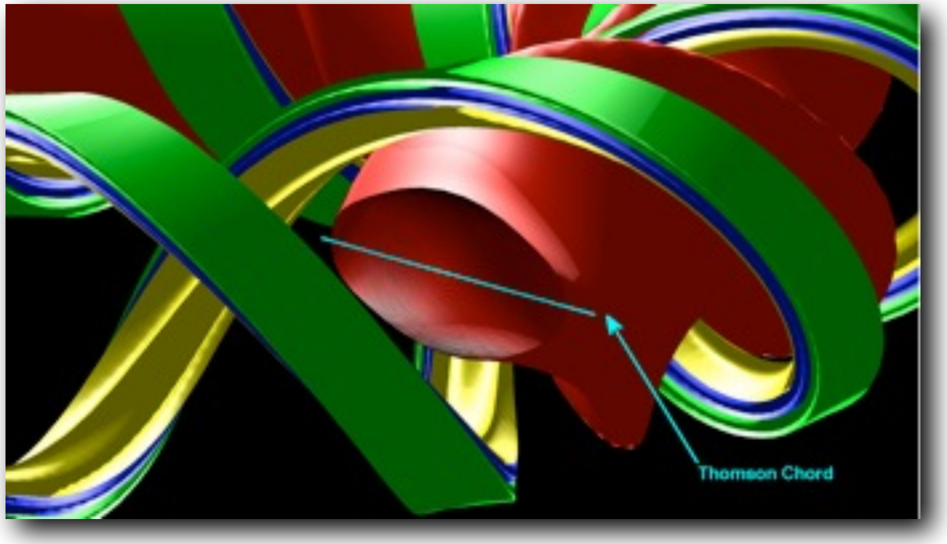
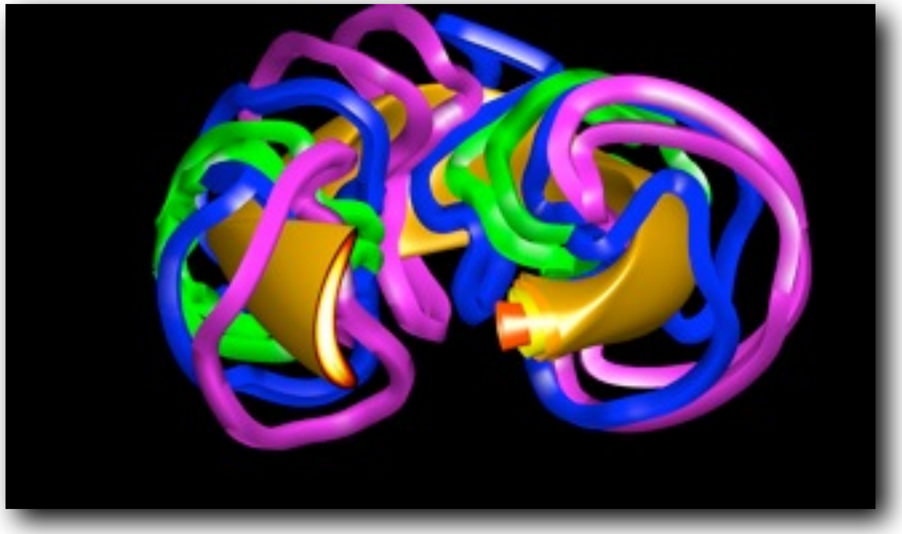
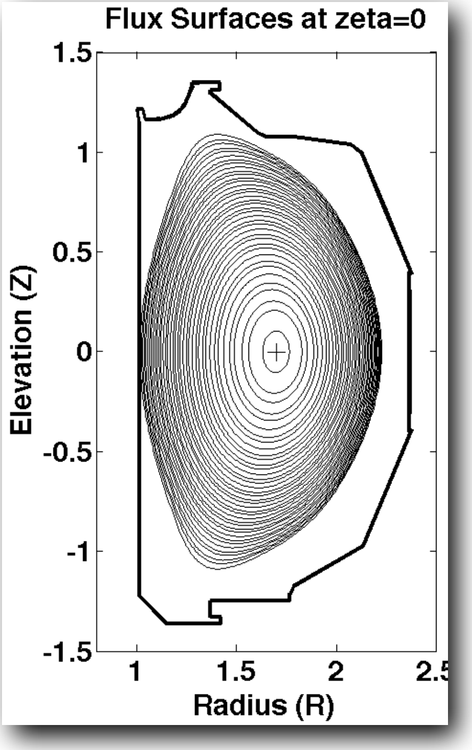
STELLOPT fits a VMEC equilibrium to targets

- Optimization Studies
  - Global Stability
  - Turbulent Transport
- Experimental Design
  - HSX, CTH, NCSX
- Reconstruction
  - W7-AS, LHD, DIII-D

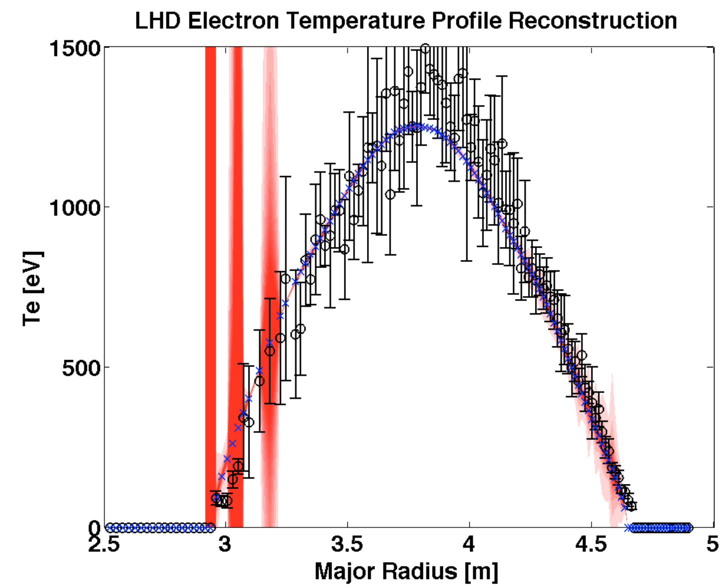


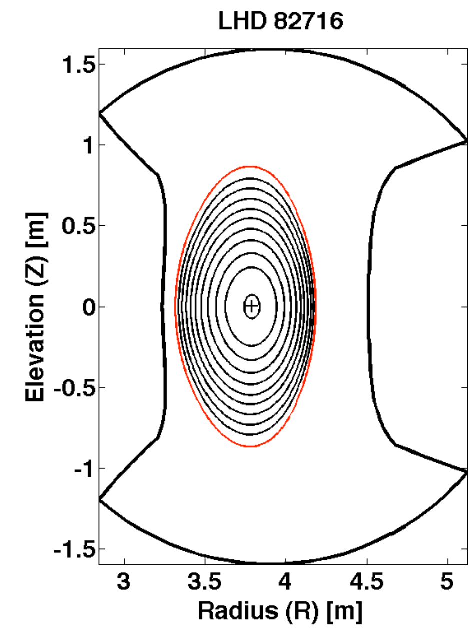


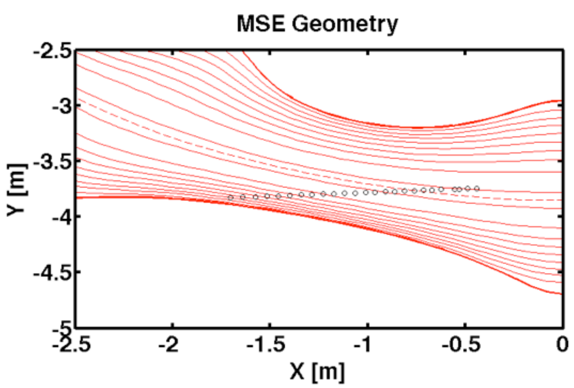


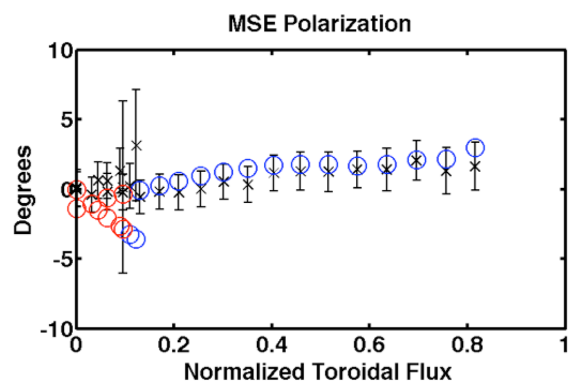
LHD Equilibrium Reconstruction

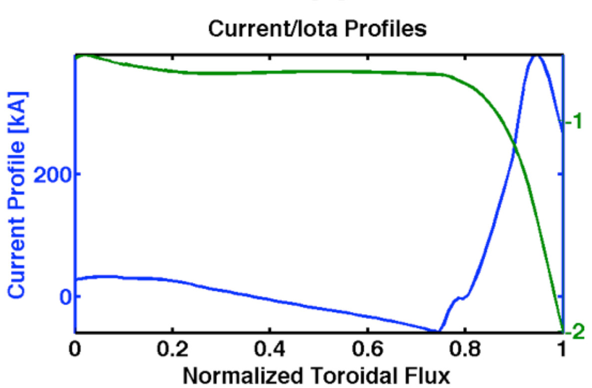
- Free boundary 3D equilibrium fit to
    - Thomson
    - Inteferrometry
    - MSE

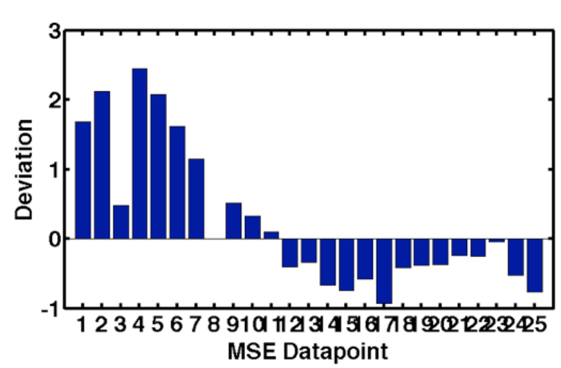


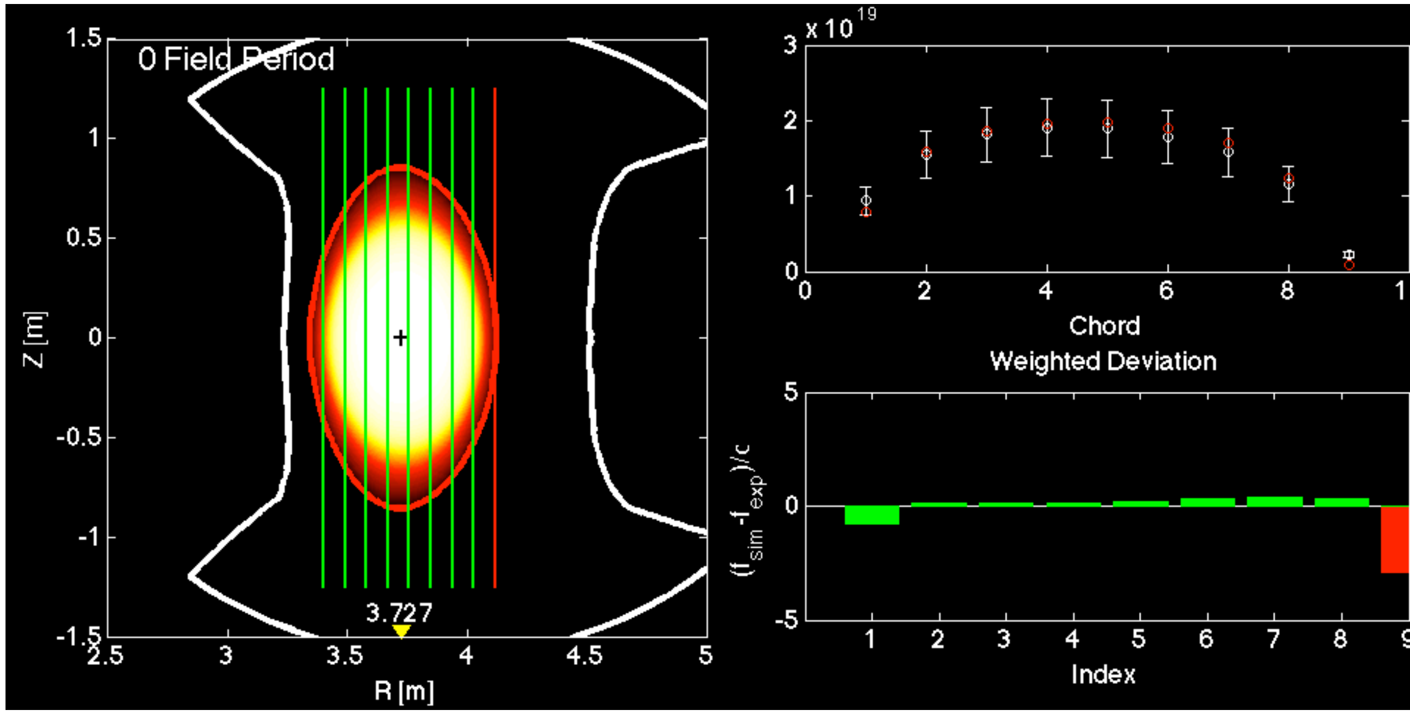






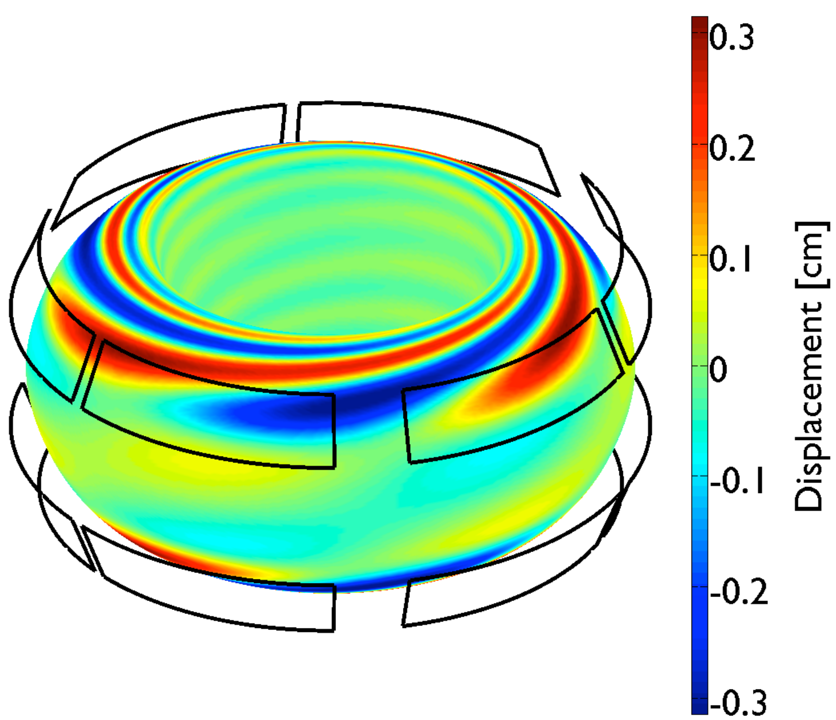


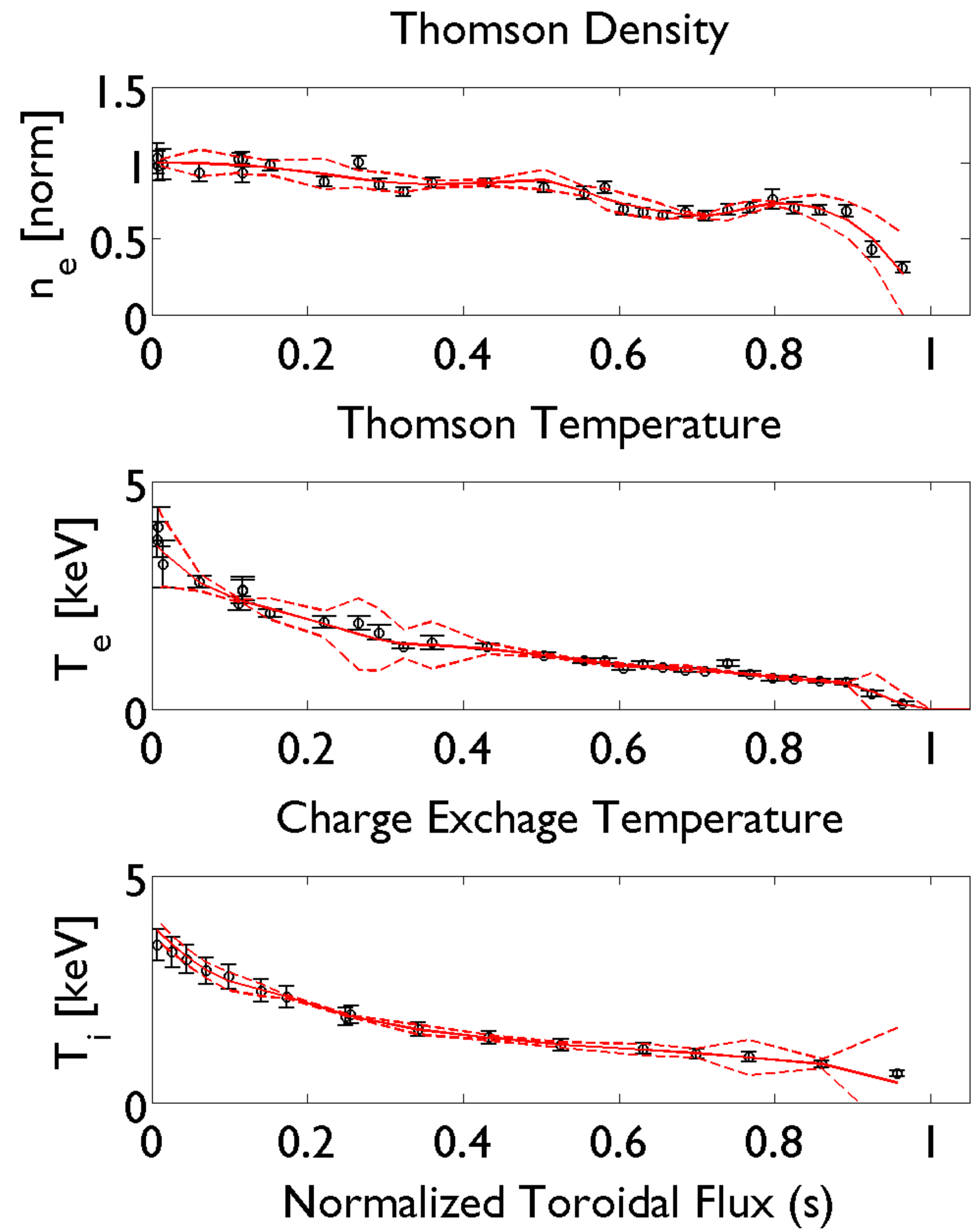




DIII-D 3D Equilibrium Reconstruction

- Free boundary 3D (n=3) equilibrium fit to
  - Magnetics
  - Thomson
  - Charge Exchange
  - Inteferrometry
  - MSE

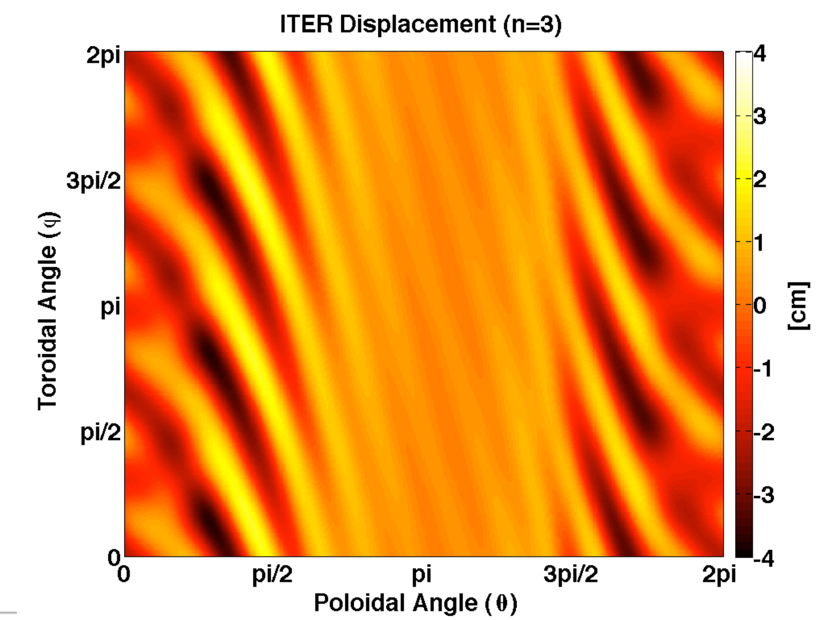


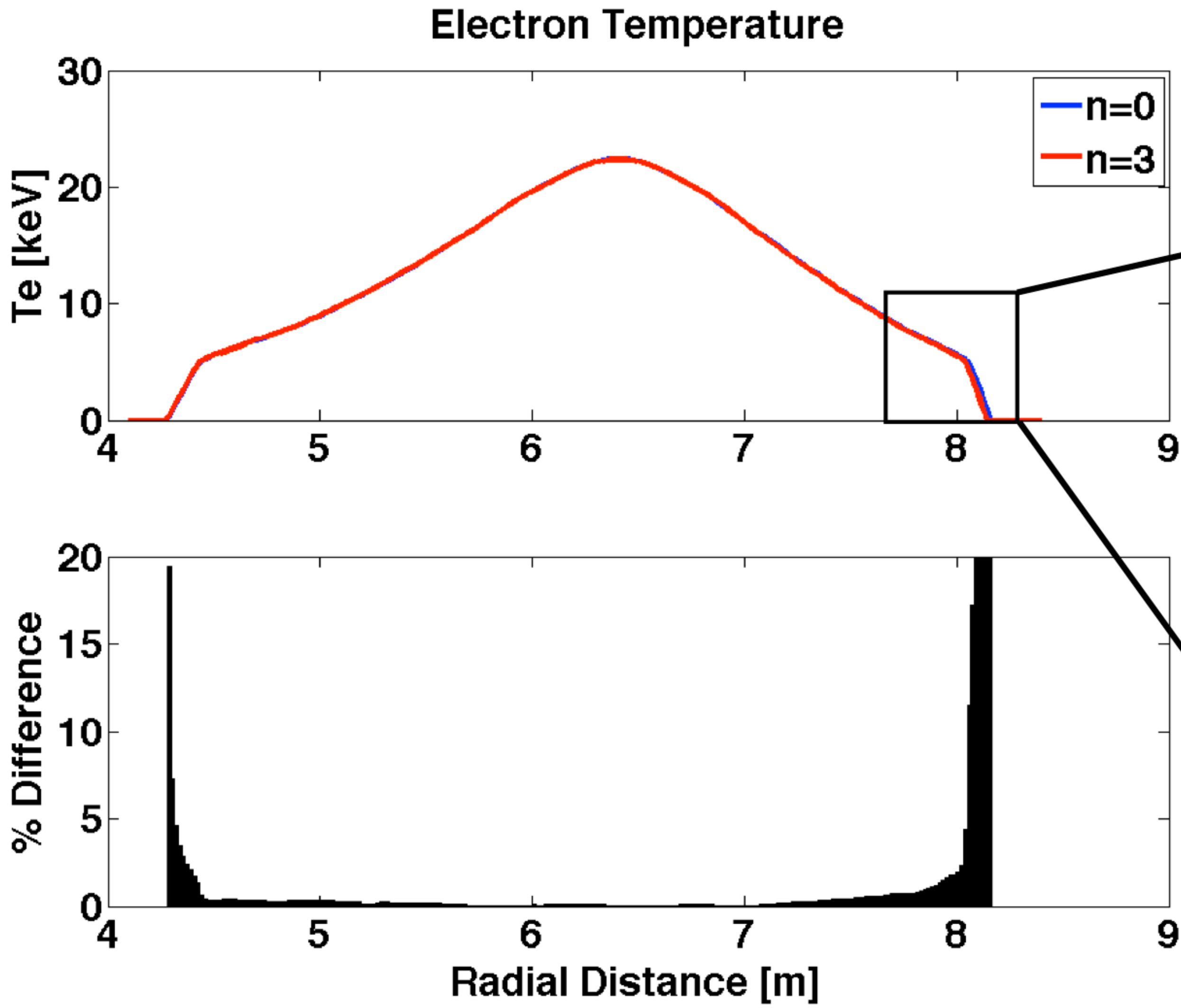


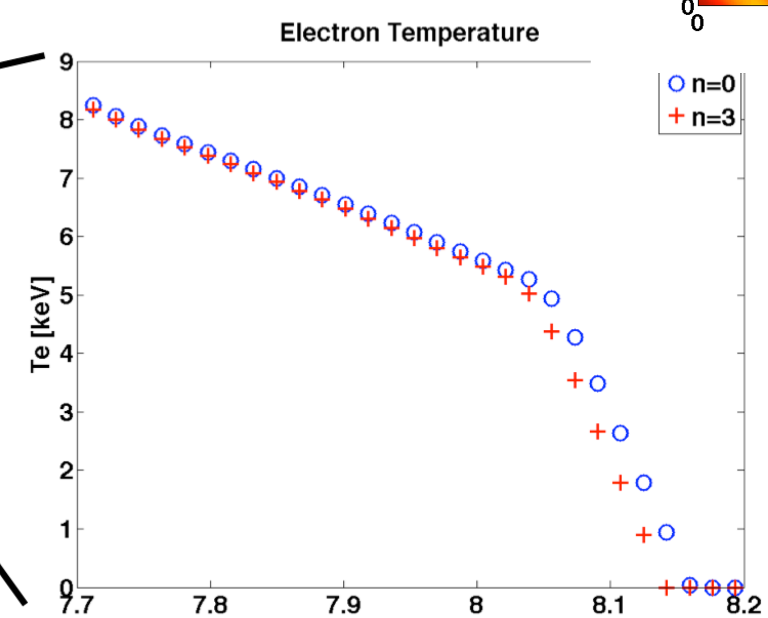
S. Lazerson, submitted to Nuclear Fusion

Forward modeling of ITER diagnostic response

- CORSICA used for transport modeling
- STELLOPT used n=3 RMP diagnostic response







~2 cm shift on outboard side

S. Lazerson et al., Plasma Phys. Control. Fusion 55 084004 (http://dx.doi.org/10.1088/0741-3335/55/8/084004)

Turbulent transport optimization of stellarators

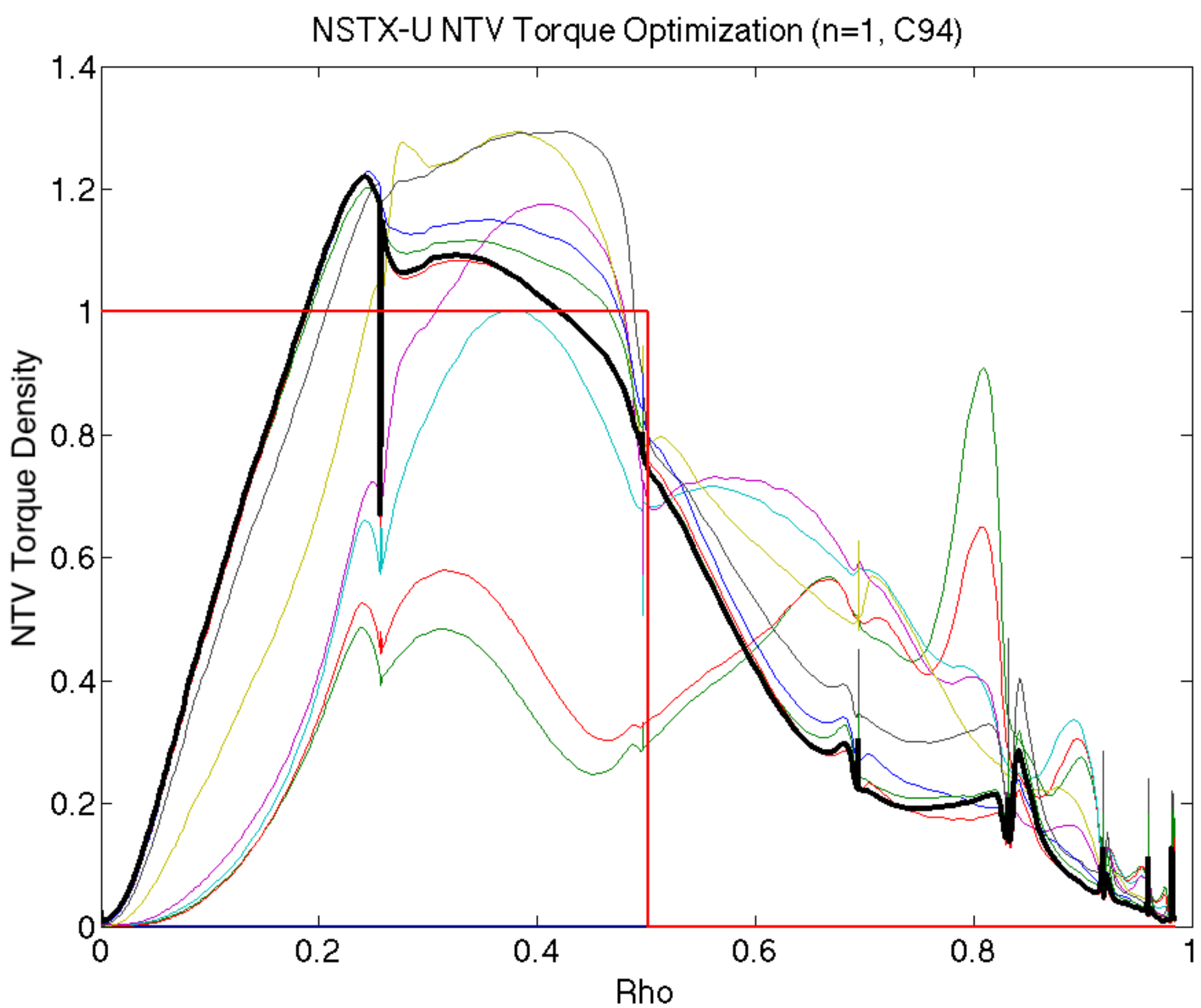
- The capability to optimize stellarators for reduced ITG and TEM turbulence has been demonstrated (Mynick, Pomphrey, Helander, Proll, Xanthopolous)
- The turbulent proxies have been included in STELLOPTV2
- Work is ongoing to incorporate parallel linear GENE into STELLOPTV2
- W7-X should allow validation

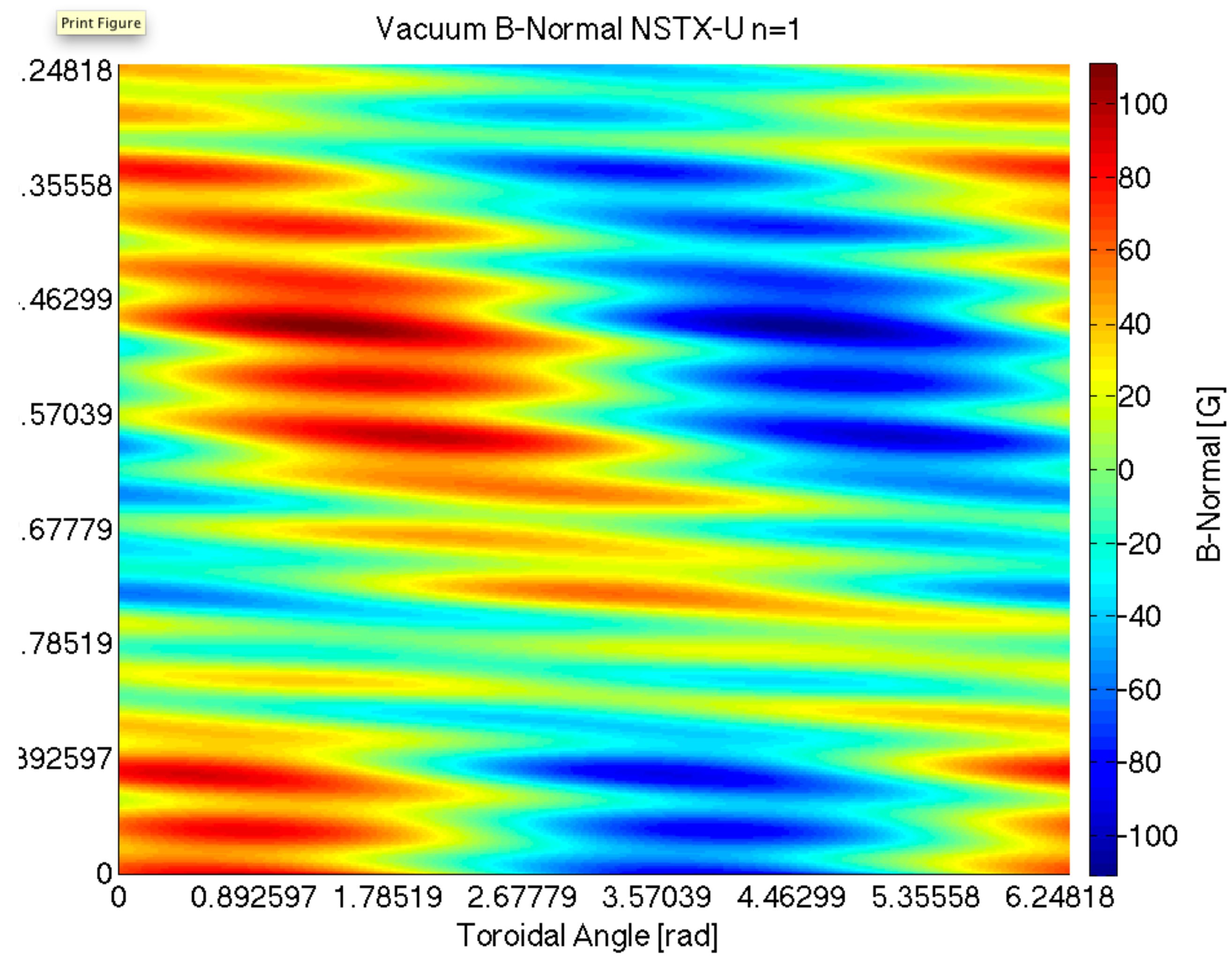
An IPEC based optimization code has been developed

- Calculates a least squares fit of IPEC input parameters to target physics parameters
    - Based on STELLOPT
    - Multiple optimization techniques
    - Targeting NTV torque as calculated by PENT
    - Fixed and free boundary optimizations
    - Coil currents can be directly optimized

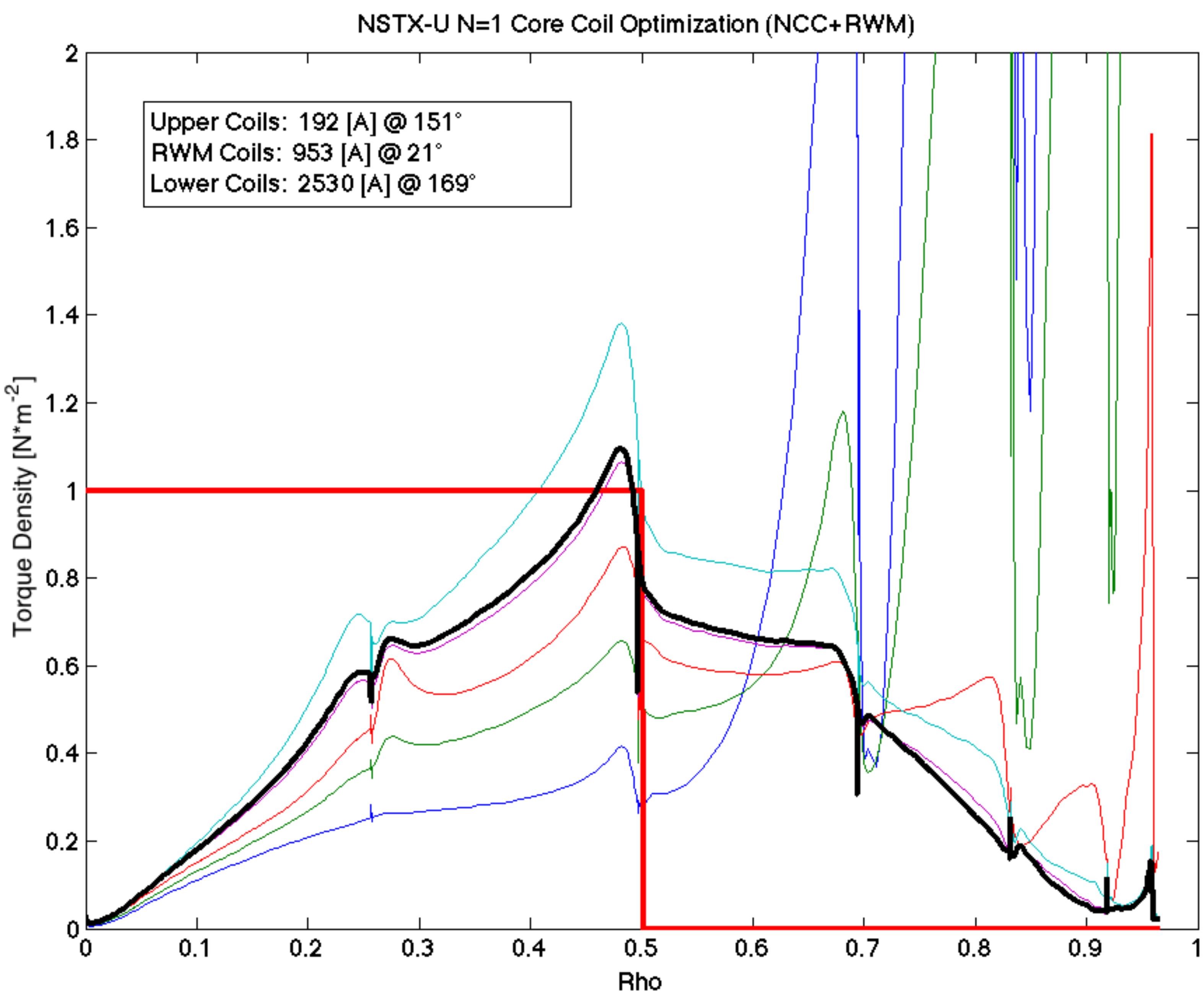
NSTX-U Core Torque Optimization

- An applied vacuum B-normal spectrum was optimized
  - Core Torque tageting (~0.5 [N.m])
  - n=1 spectrum



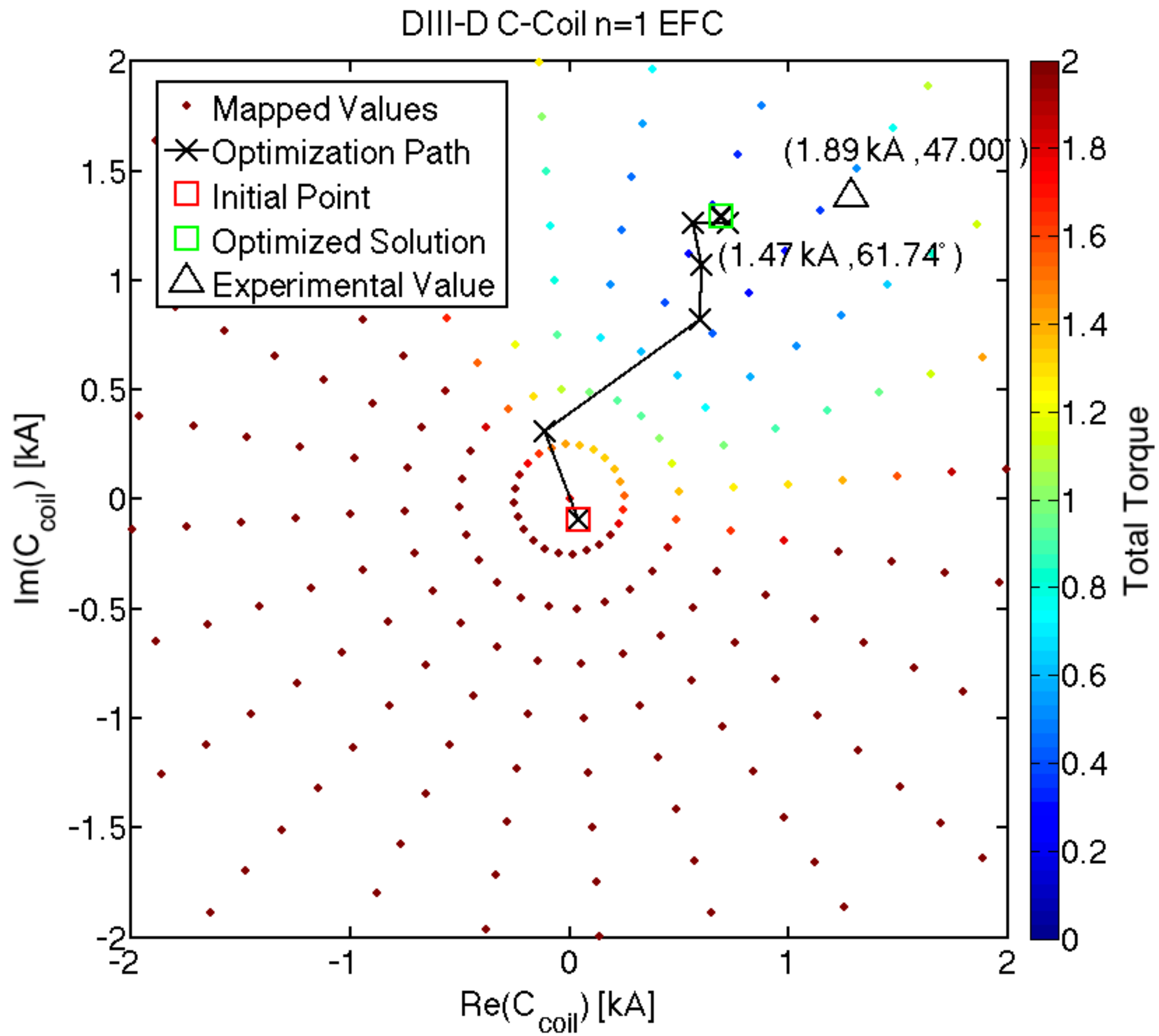


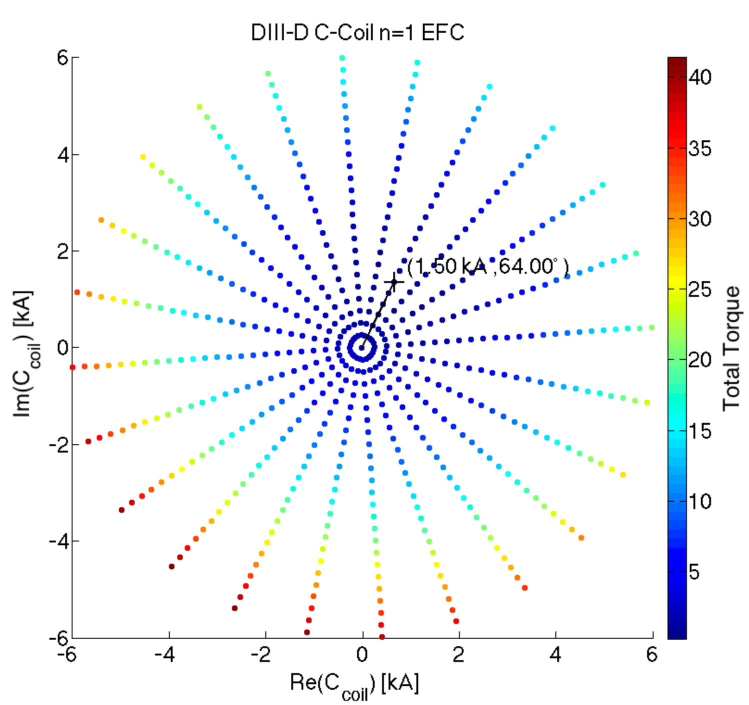
RWM+NCC coil optimization



DIII-D error field experiments used to validate

- DIII-D C-coil rotation scan experiments
    - C-coil phase and amplitude scan performed (2D parameter space)
    - Optimizer used for similar experiment (SURFMN error field)



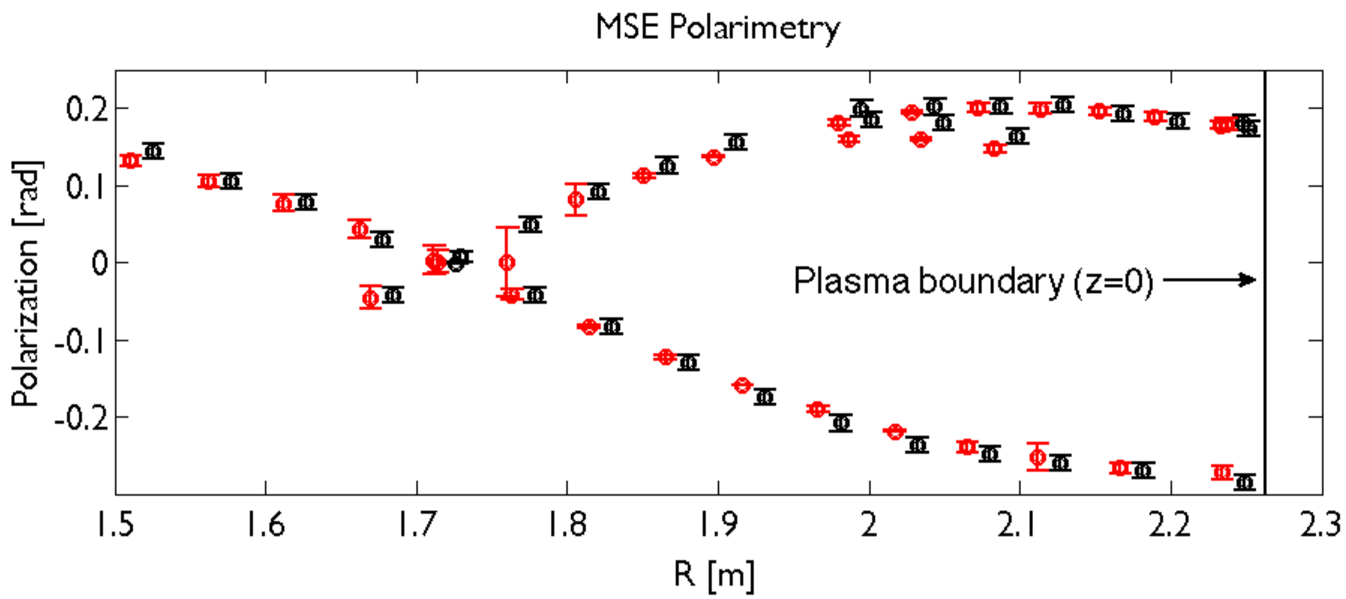


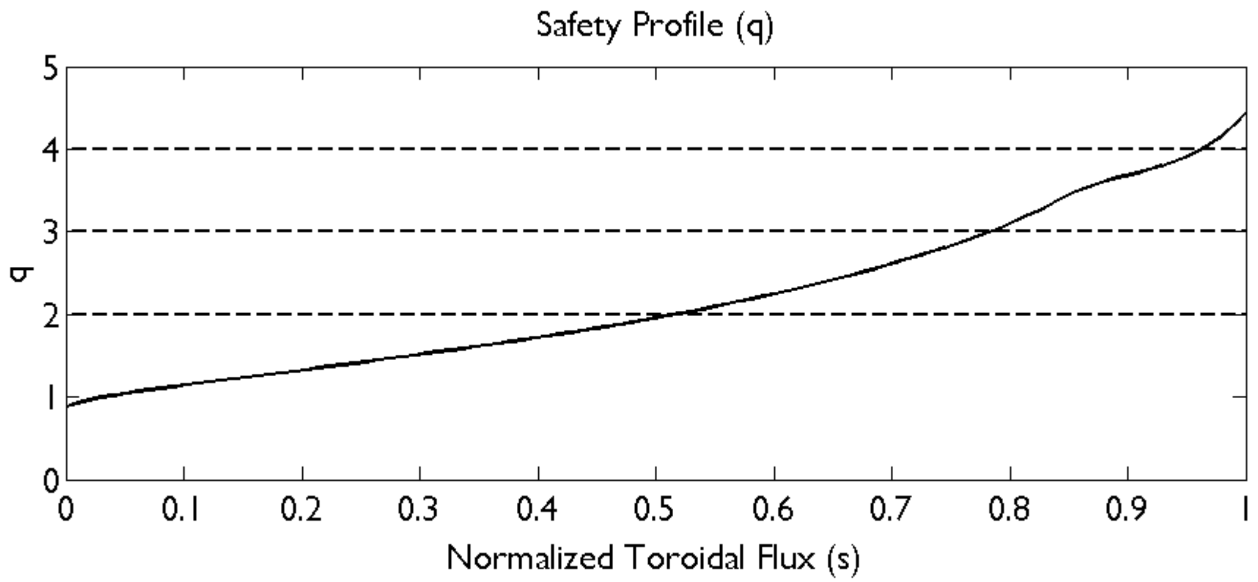
Short term plans (next year)

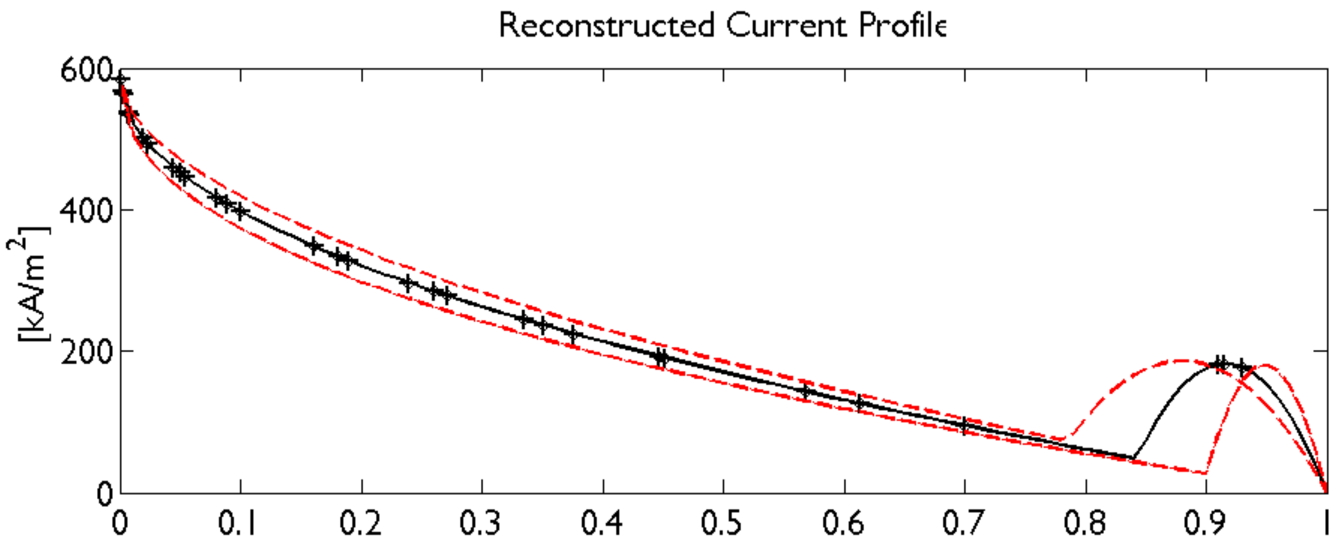
- Publish IPECOPT results (PPCF)
- Implement 3D equilibrium reconstruction on W7-X
- STELLOPT/GENE runs on Hydra/Hopper
- Energetic particle optimization in STELLOPT

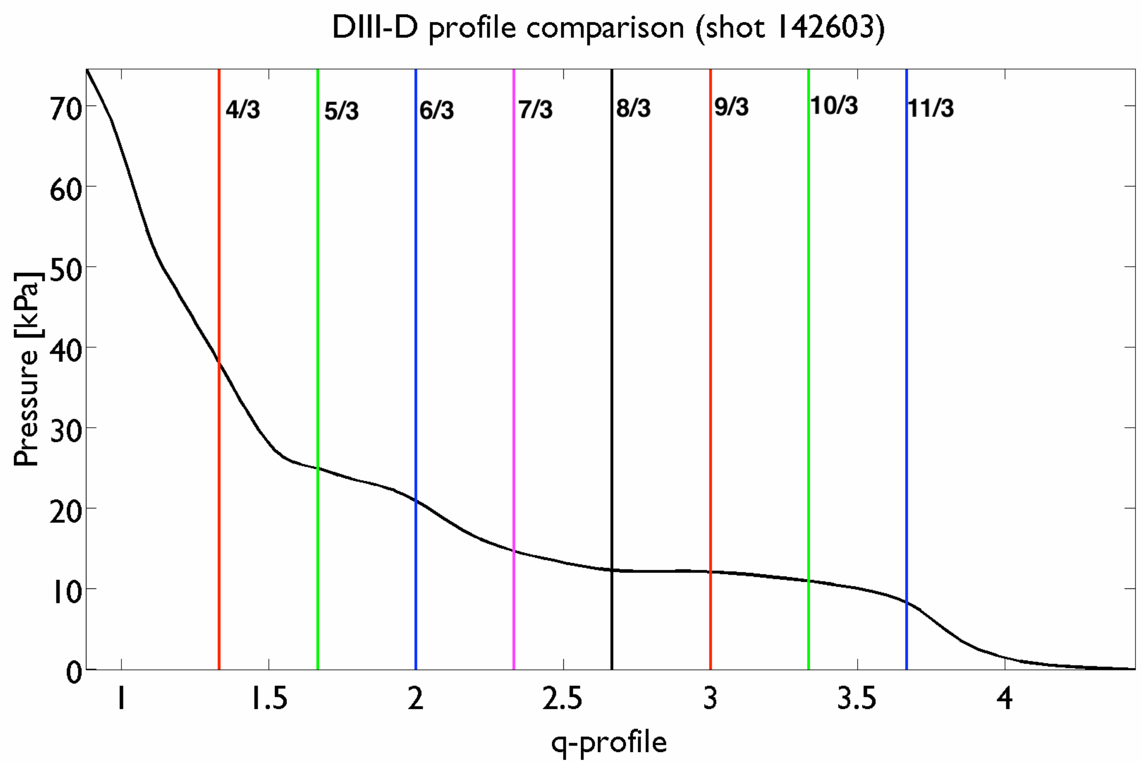
Additional Slides

DIII-D MSE Fit









DIII-D Magnetics